Some calculus concerning a sphere

(for the benefit of someone who is unable to do it himself)

The circumference of a circle is
$$2\,r\,\pi$$

To find the surface of a sphere this expression needs to be integrated along R.
$$dR = r\cos\theta$$
hence
$$2\pi\!\int r\,dR =$$
$$2\pi\!\int r^2\cos\theta\,d\theta =$$
$$2\pi\,r^2\sin\theta$$

to be evaluated from 0 to $\pi/2$
$$= 2r^2\pi$$

is the surface of a half-sphere, so $4\,r^2\,\pi$ is the surface of a sphere.

Now consider forces. An external force acts inward on every point on the surface. Say, magnitude of force in some given condition is f_1, from every direction. Thus the effect of all forces is proportional to A, say $f_1 A$.

The divergence theorem
$$\int \mathbf{f}\cdot\mathbf{n}\ dA \;=\; \int \nabla\cdot\mathbf{f}\,dV \;=\; \kappa$$

transforms the surface integral into a volume integral. $\nabla\cdot\mathbf{f}$ is the source density, and scale-independent. It follows that
$$\frac{f_1\,4\,r^2\,\pi}{\frac{4}{3}r^3\pi} = \frac{3f_1}{r}$$

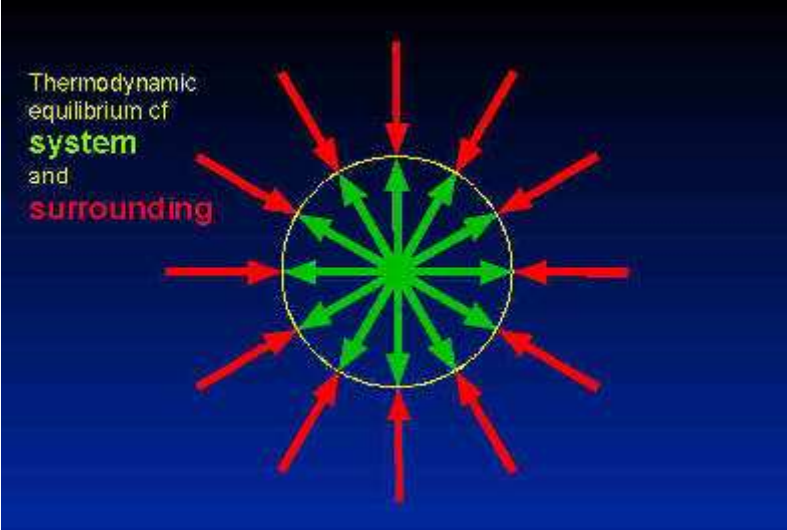is constant. Thus if V → 0, r → 0, and for the expression to remain constant it follows that

$$f_1 \sim r$$

On the other hand, we know that

$$A \sim r^2$$

hence the ratio $\frac{f}{A} \to \infty$ if V → 0.

I am not going to explain $\mathbf{f}$ or $\mathbf{f}\cdot\mathbf{n}$, or else I must assume that I also need to explain how to read and write.

In the figure below a thermodynamic system is shown. The red forces are exerted by the surrounding at the system, the green ones are exerted by the system at the surrounding. In thermodynamics, all terms must be scaled to mass (n.b. the thermodynamic mass which is dimensionless, it is measured in atoms, and not the inertial mass which is measured in kg). For this reason the divergence $\nabla \cdot \mathbf{f}$ must be scale-independent, its numerical value characterizes the energetic state in which the system is, independent of its size. If the system approaches zero, all terms connected with it – especially terms relating to work – must vanish with it.

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